Student: _____ Class: _____ Date: _____

Using Recursion in Models and Decision Making: Recursion Using Rate of Change

IV.C Student Activity Sheet 6: Rates of Change in Exponential Models

1. Consider the exponential function $y = 2^x$. Fill in the table of values for the function, and find the rate of change between consecutive values in the function (Δy). What pattern do you see for Δy change?

x	y	Δy
0		
1		
2		
3		
4		
5		

2. In Student Activity Sheet 3, you learned about the sometimes fatal antibiotic-resistant staph bacteria methicillin-resistant *Staphylococcus aureus* (MRSA) growing in an agar dish. The initial area occupied by the bacteria in the agar dish is 2 square millimeters, and they increase in area by 20% each week. The table below gives the area of the bacteria over several weeks. Use the table to describe the rate of growth of the area of the bacteria (Δa).

x (no. of weeks)	a (area in mm^2)	Δa
0	2	
1	2.4	
2	2.88	
3	3.456	
4	4.147	
5	4.977	

3. Suppose a quantity increases at a rate proportional to the quantity, and the constant of proportionality is 0.2. The initial quantity is 2. Write a difference equation that describes the statement above, and find several values of this quantity.

x	Δy	y
0		2
1		
2		
3		
4		

4. Use spreadsheet software to generate about 75 values of the table you started in Question 3. The spreadsheet allows you to use a recursive rule to generate the data.

5. The agar dish that the MRSA bacteria are growing in has an area of 1,000 square millimeters. The growth of the bacteria is limited in the lab by the size of the agar dish. The growth of the bacteria can still be modeled by a proportional difference equation, but now the rate of increase of the bacteria's area is directly proportional to the bacteria's area and the difference between the agar dish's area and the fungus's area. This constant of proportionality is the ratio of the original constant of proportionality (0.2) and the maximum area the bacteria can reach (1,000 square millimeters). The difference equation can be written as follows:

$$\frac{\Delta A}{\Delta t} = \frac{0.2}{1{,}000} A(1{,}000 - A)$$

Let $\Delta t = 1$ to simplify your work, and then use the difference equation to find the new values of A. Use a spreadsheet to calculate about 75 values.

6. Compare the data generated in the unrestricted and restricted growth models. Record your observations.

7. Use spreadsheet software or your graphing calculator to make a graph of the restricted growth model. Sketch the graph below. What observations can you make about the graph?

8. **REFLECTION**: The unrestricted bacteria growth models exponential growth and has a common ratio of 1.2. Use the spreadsheet to find the ratio between successive values in the restricted growth model. What do you notice? How does this support the graph in Question 7?

9. A rancher has decided to dedicate a 400-square-mile portion of his ranch as a black bear habitat. Working with his state, he plans to bring 10 young black bears to the habitat in an effort to grow the population. His research shows that the annual growth rate of black bears is about 0.8. Black bears thrive when the population density is no more than about 1.5 black bears per square mile.

   a. What is the maximum sustainable number of black bears for the habitat?

   b. Write a recursive rule showing the restricted growth in population for the black bears. (***Hint:*** The constant of proportionality is the ratio of the unrestricted growth rate and the maximum sustainable population.)

   c. Make a table and graph showing the yearly population of the black bears in the habitat. (Include enough years to show the population reaching the maximum sustainable population.)

   d. When will the population of bears in the habitat reach 500?

   e. The rancher wants to repopulate the state with black bears. The rancher's original plan was to release the bears from his ranch when the population reaches 500. Do you think this is a good decision based on the growth rate within the habitat over time? If you agree with the rancher, support the decision with your data and graph. If you disagree, propose a different target population value to the rancher; again, support your proposal with the data and graph.

10. **EXTENSION**: Research population data, either of humans in various parts of the world or animal species. You need to find data over a significant time, not just a few years. Cite your source. Make a scatterplot of the data. Do the data show exponential growth or do they show signs that the population's growth is slowing? What limitations does the population you are analyzing have? Could you predict a maximum population? Support your prediction.